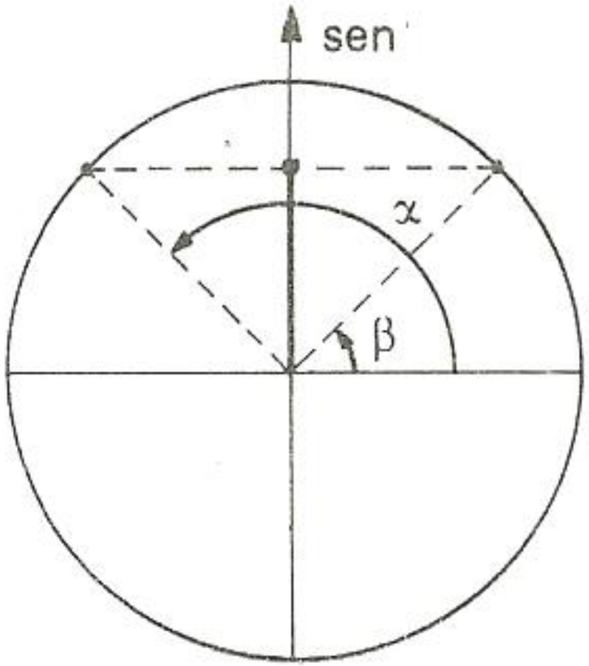
TRIGONOMETRIA

EQUAÇÕES, INEQUAÇÕES, ADIÇÃO DE ARCOS E ARCO DUPLO

PROFESSOR MARCOS JOSÉ

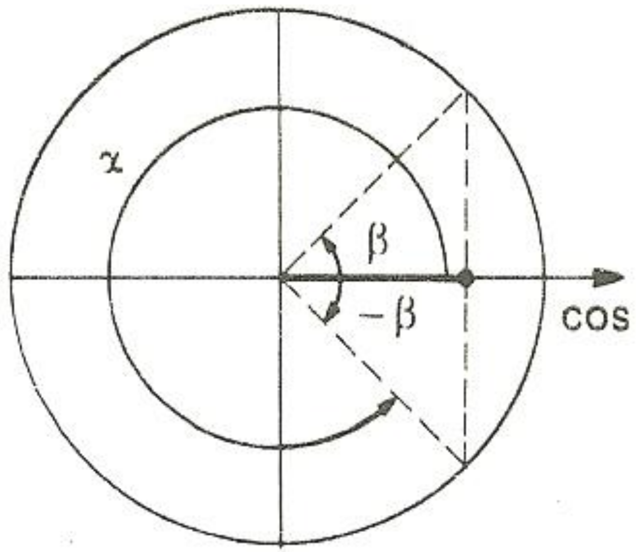
Equações Trigonométricas

1º caso: sen α = sen β



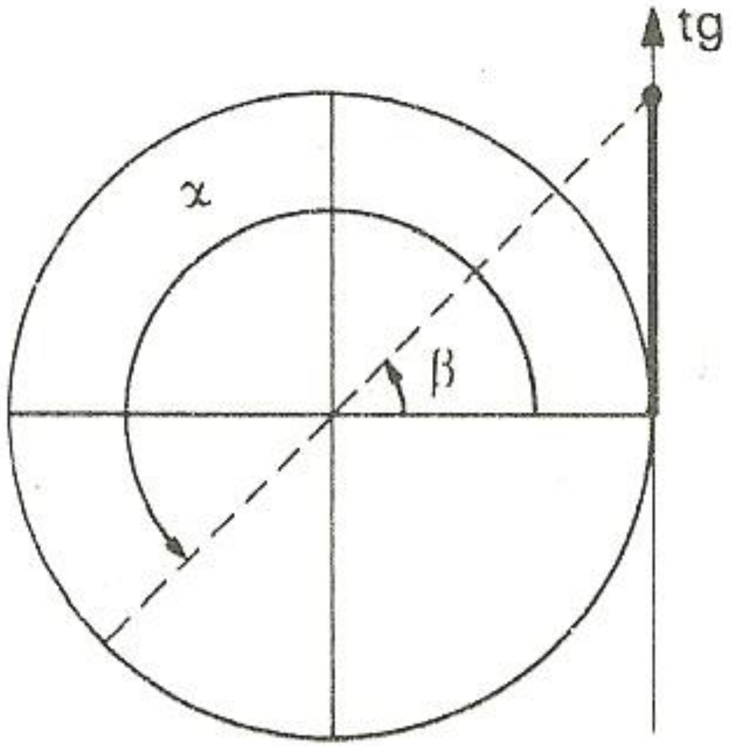
$$sen\alpha = sen\beta \rightarrow \begin{cases} \alpha = \beta + 360°.k, k \in Z \ ou \ \alpha = \beta + 2.k.\pi, k \in Z \\ \alpha = (180° - \beta) + 360°.k, k \in Z \ ou \ \alpha = (\pi - \beta) + 2.k.\pi, k \in Z \end{cases}$$

2º caso: cos α = cos β



$$\cos\alpha = cos\beta \rightarrow \begin{cases} \alpha = \beta + 360°.k, k \in Z \ ou \ \alpha = \beta + 2.k.\pi, k \in Z \\ \alpha = (360° - \beta) + 360°.k, k \in Z \ ou \ \alpha = (2\pi - \beta) + 2.k.\pi, k \in Z \end{cases}$$

3º caso: tg α = tg β



$$\text{tg}\alpha = tg\beta \rightarrow \begin{cases} \alpha = \beta + 360°.k, k \in Z \ ou \ \alpha = \beta + 2.k.\pi, k \in Z \\ \alpha = (180° + \beta) + 360°.k, k \in Z \ ou \ \alpha = (\pi + \beta) + 2.k.\pi, k \in Z \end{cases}$$

OBS.: $\alpha = \beta + 180°.k, k \in Z \ ou \ \alpha = \beta + k.\pi, k \in Z$

Exercícios

1) Resolva as equações abaixo:

$$a)\ senx = \frac{1}{2}$$

$$\boldsymbol{senx = sen30° \rightarrow senx > 0 \rightarrow x \in 1°\ quadrante\ ou\ x \in 2°\ quadrante}$$

$$\begin{cases} \boldsymbol{x = 30° + 360°.k, k \in Z} \\ \boldsymbol{x = (180° - 30°) \rightarrow x = 150° + 360°.k, k \in Z} \end{cases}$$

$$\boldsymbol{Geralmente, a\ resolução\ de\ equações\ tem\ a\ resposta\ em\ radianos.}$$

$$\begin{cases} \boldsymbol{x = \frac{\pi}{6} + 2.k.\pi, k \in Z} \\ \boldsymbol{x = \left(\pi - \frac{\pi}{6}\right) \rightarrow x = \frac{5\pi}{6} + 2.k.\pi, k \in Z} \end{cases}$$

$b)\ cosx = -\frac{\sqrt{3}}{2}$

$\boldsymbol{cosx < 0 \rightarrow x \in 2° \ quadrante \ ou \ x \in 3° \ quadrante}$

$\boldsymbol{temos \ que, no \ primeiro \ quadrante, cos30° = \frac{\sqrt{3}}{2}}$

$$\begin{cases} x = (180° - 30°) \rightarrow x = 150° + 360°.k, k \in Z \\ x = (180° + 30°) \rightarrow x = 210° + 360°.k, k \in Z \end{cases}$$

$$\begin{cases} x = \left(\pi - \frac{\pi}{6}\right) \rightarrow x = \frac{5\pi}{6} + 2.k.\pi, k \in Z \\ x = \left(\pi + \frac{\pi}{6}\right) \rightarrow x = \frac{7\pi}{6} + 2.k.\pi, k \in Z \end{cases}$$

$c)\ tgx = -\sqrt{3}$

$tgx < 0 \rightarrow x \in 2° \text{ quadrante ou } x \in 4° \text{ quadrante}$

$\text{temos que, no primeiro quadrante, } tg\,60° = \sqrt{3}$

$$\begin{cases} x = (180° - 60°) \rightarrow x = 120° + 360°.k,\ k \in Z \\ x = (360° - 60°) \rightarrow x = 300° + 360°.k,\ k \in Z \end{cases}$$

$$\begin{cases} x = \left(\pi - \frac{\pi}{3}\right) \rightarrow x = \frac{2\pi}{3} + 2.k.\pi,\ k \in Z \\ x = \left(2\pi - \frac{\pi}{3}\right) \rightarrow x = \frac{5\pi}{3} + 2.k.\pi,\ k \in Z \end{cases}$$

$$\text{No caso da tangente} \rightarrow x = 120° + 180°.k \text{ ou } x = \frac{2\pi}{3} + k.\pi,\ k \in Z$$

$$d)\ senx = -\frac{\sqrt{2}}{2}, com\ 0° < x < 360°$$

$$\boldsymbol{senx < 0 \rightarrow x \in 3°\ quadrante\ ou\ x \in 4°\ quadrante}$$

$$\boldsymbol{temos\ que, no\ primeiro\ quadrante, sen45° = \frac{\sqrt{2}}{2}}$$

$$\begin{cases} x = (180° + 45°) \rightarrow x = 225° + 360°.k, k \in Z \\ x = (360° - 45°) \rightarrow x = 315° + 360°.k, k \in Z \end{cases}$$

$$\boldsymbol{como\ 0° < x < 360° \rightarrow x = 225°\ ou\ x = 315°}$$

$$\boldsymbol{S = \{225°, 315°\}}$$

$e)\ sen^2x + 2.senx - 3 = 0$

$$\boldsymbol{senx = t}$$

$$\boldsymbol{t^2 + 2t - 3 = 0 \rightarrow t = \frac{-2 \pm \sqrt{(2)^2 - 4.1.(-3)}}{2.1} \rightarrow t = \frac{-2 \pm \sqrt{16}}{2} \rightarrow t = \frac{-2 \pm 4}{2}}$$

$$\begin{cases} \boldsymbol{t_1 = \frac{-2+4}{2} = 1} \\ \boldsymbol{t_2 = \frac{-2-4}{2} = -3} \end{cases}$$

$$\begin{cases} \boldsymbol{senx = 1 \rightarrow x = 90° + 360°.k, k \in Z\ ou\ x = \frac{\pi}{2} + 2.k.\pi, k \in Z} \\ \boldsymbol{senx = -3 \rightarrow não\ existe, pois\ -1 \leq senx \leq 1} \end{cases}$$

$$\boldsymbol{S = \{x = \frac{\pi}{2} + 2.k.\pi, k \in Z\}}$$

$f)\ secx + 2.cosx = 0$

$$\frac{1}{cosx} + 2.cosx = 0 \rightarrow 1 + 2.cos^2x = 0 \rightarrow 2.cos^2x = -1 \rightarrow cos^2x = -\frac{1}{2}$$

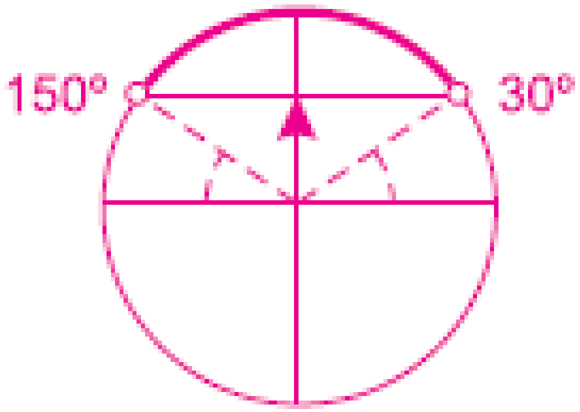
$S = \emptyset$, ***pois não existe número real que, ao quadrado, seja negativo***

2) Resolva as inequações abaixo.

$a)\ 2.senx\ -1\ > 0, com\ 0°\ < x < 360°$

$\mathbf{2.senx - 1 > 0 \rightarrow 2.senx > 1 \rightarrow senx > \frac{1}{2}}$

$\mathbf{senx = \frac{1}{2} \rightarrow x = 30°\ ou\ x = 150°}$

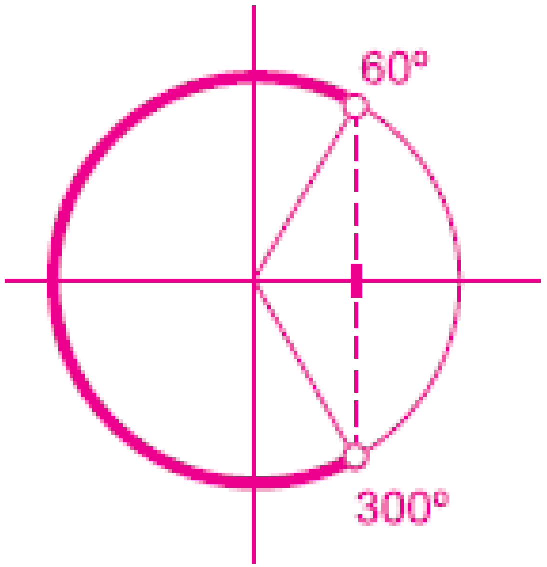


$\mathbf{S = \{x \in R; 30° < x < 150°\}}$

$b)\ 2.cosx - 1 < 0, com\ 0° < x < 360°$

$$\mathbf{2.cosx - 1 < 0 \rightarrow 2.cosx < 1 \rightarrow cosx < \frac{1}{2}}$$

$$\mathbf{cosx = \frac{1}{2} \rightarrow x = 60°\ ou\ x = 300°}$$

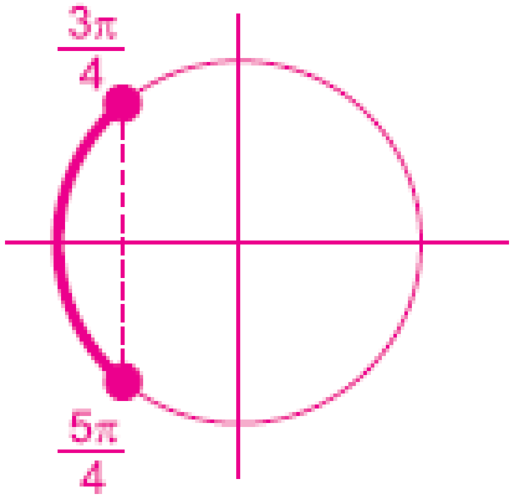


$$\mathbf{S = \{x \in R; 60° < x < 300°\}}$$

$c)\ \sqrt{2}.cosx + 1 \leq 0, com\ 0 \leq x \leq 2\pi$

$$\sqrt{2}.cosx + 1 \leq 0 \rightarrow \sqrt{2}.cosx \leq -1 \rightarrow cosx \leq -\frac{1}{\sqrt{2}} \rightarrow cosx \leq -\frac{\sqrt{2}}{2}$$

$$cosx = -\frac{\sqrt{2}}{2} \rightarrow x = 135° = \frac{3\pi}{4}\ ou\ x = 225° = \frac{5\pi}{4}$$

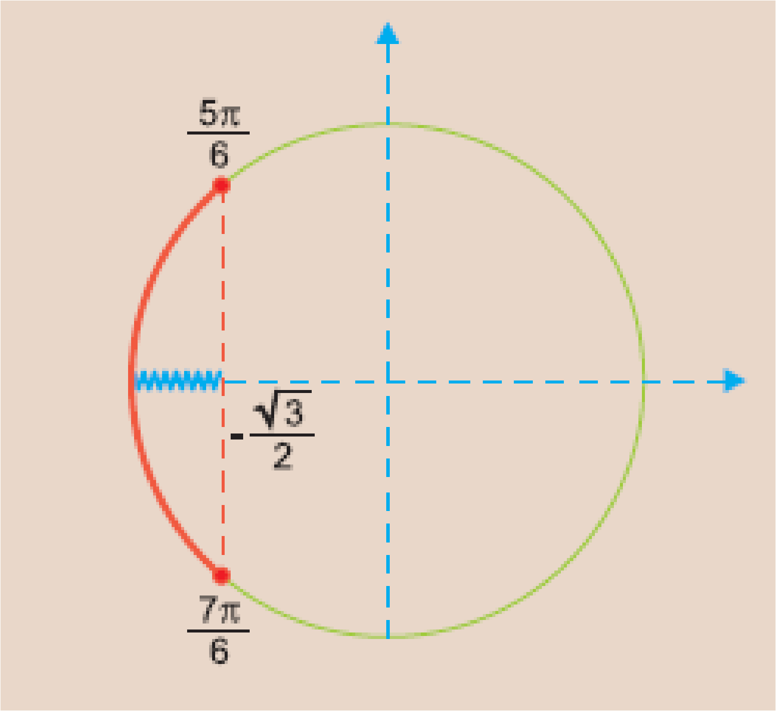


$$S = \left\{x \in R;\ \frac{3\pi}{4} \leq x \leq \frac{5\pi}{4}\right\}$$

$d)\ 2.cosx + \sqrt{3} \leq 0, com\ 0 \leq x \leq 2\pi$

$$\mathbf{2.cosx + \sqrt{3} \leq 0 \rightarrow 2.cosx \leq -\sqrt{3} \rightarrow cosx \leq -\frac{\sqrt{3}}{2}}$$

$$\mathbf{cosx = \frac{\sqrt{3}}{2} \rightarrow x = 150° = \frac{5\pi}{6}\ ou\ x = 210° = \frac{7\pi}{6}}$$

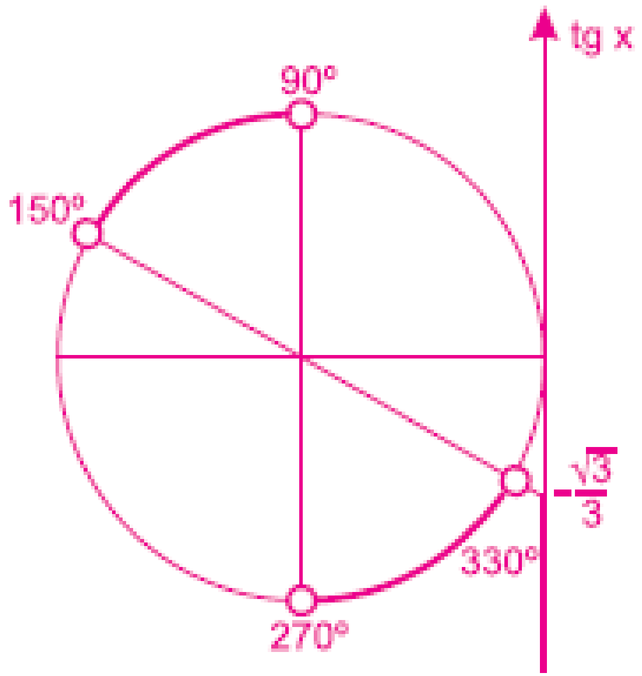


$$\mathbf{S = \left\{x \in R; \frac{5\pi}{6} \leq x \leq \frac{7\pi}{6}\right\}}$$

$e)\ 3.tgx + \sqrt{3} < 0, com\ 0° < x < 360°$

$$\mathbf{3.tgx + \sqrt{3} < 0 \rightarrow 3.tgx < -\sqrt{3} \rightarrow tgx < -\frac{\sqrt{3}}{3}}$$

$$\mathbf{tgx = -\frac{\sqrt{3}}{3} \rightarrow x = 150°\ ou\ x = 330°}$$



$$\mathbf{S = \{x \in R; 90° < x < 150°\ ou\ 270° < x < 330°\}}$$

ADIÇÃO E SUBTRAÇÃO DE ARCOS

1.

$sen(a + b) = sen\ a . cos\ b + sen\ b . cos\ a$
$sen(a - b) = sen\ a . cos\ b - sen\ b . cos\ a$

$\forall a, b \in \mathbb{R}$

2.

$cos(a + b) = cos\ a . cos\ b - sen\ a . sen\ b$
$cos(a - b) = cos\ a . cos\ b + sen\ a . sen\ b$

$\forall a, b \in \mathbb{R}$

3.

$tg(a + b) = \dfrac{tg\ a + tg\ b}{1 - tg\ a\ .\ tg\ b}$
$tg(a - b) = \dfrac{tg\ a - tg\ b}{1 + tg\ a\ .\ tg\ b}$

(supondo que **a**, **b**, **a + b** e **a − b** sejam, todos, diferentes de $\frac{\pi}{2} + n\pi$, com $n \in \mathbb{Z}$)

ARCO DUPLO

$$\operatorname{sen}(2a) = 2 \,.\, \operatorname{sen} a \,.\, \cos a$$

$$\cos(2a) = \cos^2 a - \operatorname{sen}^2 a$$

$$\operatorname{tg}(2a) = \frac{2 \,.\, \operatorname{tg} a}{1 - \operatorname{tg}^2 a}$$

Exercícios

1) Calcule:

$a)\ sen\ 15°$

$$sen\ 15° = sen\ (45° - 30°) \rightarrow sen15° = sen45°.cos30° - sen30°.cos45°$$

$$sen15° = \frac{\sqrt{2}}{2}.\frac{\sqrt{3}}{2} - \frac{1}{2}.\frac{\sqrt{2}}{2} \rightarrow sen15° = \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} \rightarrow sen15° = \frac{\sqrt{6}-\sqrt{2}}{4}$$

$b)\ tg105°$

$$tg105° = tg(60° + 45°) \rightarrow tg105° = \frac{tg60° + tg45°}{1 - tg60°.tg45°} \rightarrow tg105° = \frac{\sqrt{3}+1}{1-\sqrt{3}.1}$$

$$tg105° = \frac{\sqrt{3}+1}{1-\sqrt{3}} x \frac{1+\sqrt{3}}{1+\sqrt{3}} \rightarrow tg105° = \frac{\sqrt{3}+3+1+\sqrt{3}}{1^2-\left(\sqrt{3}\right)^2} \rightarrow tg105° = \frac{4+2\sqrt{3}}{1-3}$$

$$tg105° = \frac{4+2\sqrt{3}}{-2} \rightarrow tg105° = -2-\sqrt{3}$$

$c)\ cos75°$

$$cos75° = cos(45° + 30°) \rightarrow cos75° = cos45°.cos30° - sen45°.sen30°$$

$$cos75° = \frac{\sqrt{2}}{2}.\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}.\frac{1}{2} \rightarrow cos75° = \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} \rightarrow cos75° = \frac{\sqrt{6} - \sqrt{2}}{4}$$

$d)\ sen75°$

$$sen75° = sen(45° + 30°) \rightarrow sen75° = sen45°.cos30° + sen30°.cos45°$$

$$sen75° = \frac{\sqrt{2}}{2}.\frac{\sqrt{3}}{2} + \frac{1}{2}.\frac{\sqrt{2}}{2} \rightarrow sen75° = \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} \rightarrow sen75° = \frac{\sqrt{6} + \sqrt{2}}{4}$$

2) Se $tg\alpha = \frac{4}{3}$ e $tg\beta = 7$, com α e β agudos, calcular $\alpha + \beta$.

$$tg(\alpha+\beta) = \frac{tg\alpha + tg\beta}{1 - tg\alpha . tg\beta} \rightarrow tg(\alpha+\beta) = \frac{\frac{4}{3} + 7}{1 - \frac{4}{3} . 7} \rightarrow tg(\alpha+\beta) = \frac{\frac{25}{3}}{1 - \frac{28}{3}}$$

$$tg(\alpha+\beta) = \frac{\frac{25}{3}}{-\frac{25}{3}} \rightarrow tg(\alpha+\beta) = -1 \rightarrow \begin{cases} \alpha+\beta = 135° \\ \alpha+\beta = 315° \end{cases}$$

$$Como\ \alpha\ e\ \beta\ são\ agudos, então\ \alpha + \beta = 135°$$

3) A expressão $\frac{\cos(\frac{\pi}{2}+x)}{sen(\frac{\pi}{2}-x)}$ é equivalente a

a) tgx b) cotgx c) –tgx d) -cotgx

$$\cos\left(\frac{\pi}{2}+x\right) = \cos\frac{\pi}{2}.\cos x - sen\frac{\pi}{2}.senx \rightarrow \cos\left(\frac{\pi}{2}+x\right) = 0.\cos x - 1.senx \rightarrow \cos\left(\frac{\pi}{2}+x\right) = -senx$$

$$sen\left(\frac{\pi}{2}-x\right) = sen\frac{\pi}{2}.\cos x - senx.\cos\frac{\pi}{2} \rightarrow sen\left(\frac{\pi}{2}-x\right) = 1.\cos x - senx.0 \rightarrow sen\left(\frac{\pi}{2}-x\right) = \cos x$$

$$\frac{\cos\left(\frac{\pi}{2}+x\right)}{sen\left(\frac{\pi}{2}-x\right)} = \frac{-senx}{\cos x} = -tgx$$

$GABARITO: C$

4) Sendo $sen\alpha = \frac{2}{3}$ e $cos\alpha = \frac{\sqrt{5}}{3}$, obter $sen(2\alpha), \cos(2\alpha)$ e $\text{tg}(2\alpha)$.

$$sen(2\alpha) = 2.sen\alpha.cos\alpha \rightarrow sen(2\alpha) = 2.\frac{2}{3}.\frac{\sqrt{5}}{3} \rightarrow sen(2\alpha) = \frac{4.\sqrt{5}}{9}$$

$$cos(2\alpha) = cos^2\alpha - sen^2\alpha \rightarrow cos(2\alpha) = \left(\frac{\sqrt{5}}{3}\right)^2 - \left(\frac{2}{3}\right)^2 \rightarrow cos(2\alpha) = \frac{5}{9} - \frac{4}{9} \rightarrow cos(2\alpha) = \frac{1}{9}$$

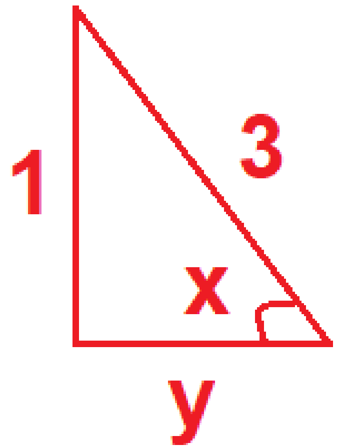
$$tg\alpha = \frac{sen\alpha}{cos\alpha} \rightarrow tg\alpha = \frac{\frac{2}{3}}{\frac{\sqrt{5}}{3}} \rightarrow tg\alpha = \frac{2}{3}.\frac{3}{\sqrt{5}} \rightarrow tg\alpha = \frac{2}{\sqrt{5}}x\frac{\sqrt{5}}{\sqrt{5}} \rightarrow tg\alpha = \frac{2\sqrt{5}}{5}$$

$$tg(2\alpha) = \frac{2tg\alpha}{1 - tg^2\alpha} \rightarrow tg(2\alpha) = \frac{\frac{2.(2\sqrt{5})}{5}}{1 - \left(\frac{2\sqrt{5}}{5}\right)^2} \rightarrow tg(2\alpha) = \frac{\frac{4\sqrt{5}}{5}}{1 - \frac{20}{25}} \rightarrow tg(2\alpha) = \frac{\frac{4\sqrt{5}}{5}}{\frac{5}{25}} \rightarrow tg(2\alpha) = \frac{4\sqrt{5}}{5}.\frac{25}{5}$$

$$tg(2\alpha) = 4\sqrt{5}$$

5) *Sendo* $0 < x < \frac{\pi}{2}$ *e senx* $= \frac{1}{3}$, *calcule sen* $(2x)$ *e* $\cos(2x)$.

$$0 < x < \frac{\pi}{2} \rightarrow 1^{\text{o}}\ quadrante$$



$$3^2 = 1^2 + y^2 \rightarrow 9 = 1 + y^2 \rightarrow y^2 = 8 \rightarrow y = 2\sqrt{2}$$

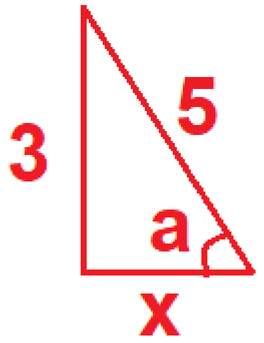
$$cosx = \frac{2\sqrt{2}}{3}$$

$$sen(2x) = 2.senx.cosx \rightarrow sen(2x) = 2.\frac{1}{3}.\frac{2.\sqrt{2}}{3} = \frac{4\sqrt{2}}{9}$$

$$cos(2x) = cos^2x - sen^2x \rightarrow cos(2x) = \left(\frac{2\sqrt{2}}{3}\right)^2 - \left(\frac{1}{3}\right)^2 \rightarrow cos(2x) = \frac{8}{9} - \frac{1}{9} = \frac{7}{9}$$

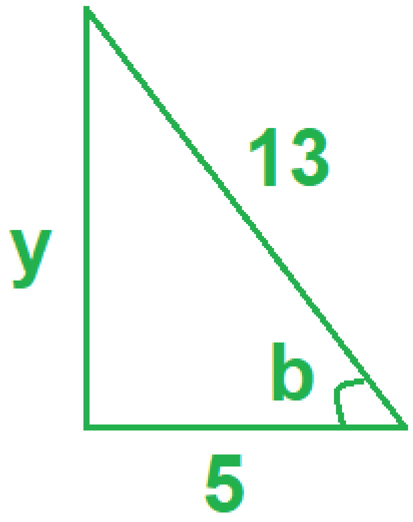
6) $Sendo\ sena = \frac{3}{5}, cosb = \frac{5}{13}, a\ e\ b\ pertencentes\ ao\ primeiro\ quadrante, calcule:$

$a)\ sen(a+b);\ b)\ \cos(a-b)\ ;\ c)\ sen(2a);\ d)\ \cos(2b)\ ;\ e)\ \text{tg}(2a)$



$$5^2 = 3^2 + x^2 \rightarrow 25 = 9 + x^2 \rightarrow x^2 = 16 \rightarrow x = 4$$

$$\begin{cases} sena = \frac{3}{5} \\ cosa = \frac{4}{5} \\ tga = \frac{3}{4} \end{cases}$$



$$13^2 = 5^2 + y^2 \rightarrow 169 = 25 + y^2 \rightarrow y^2 = 144 \rightarrow y = 12$$

$$\begin{cases} senb = \frac{12}{13} \\ cosb = \frac{5}{13} \\ tgb = \frac{12}{5} \end{cases}$$

$$\begin{cases} sena = \frac{3}{5} \\ cosa = \frac{4}{5} \\ tga = \frac{3}{4} \end{cases} \qquad \begin{cases} senb = \frac{12}{13} \\ cosb = \frac{5}{13} \\ tgb = \frac{12}{5} \end{cases}$$

$$a)\ sen(a+b) = sena.cosb + senb.cosa \rightarrow sen(a+b) = \frac{3}{5}.\frac{5}{13} + \frac{12}{13}.\frac{4}{5} \rightarrow sen(a+b) = \frac{15}{65} + \frac{48}{65} = \frac{63}{65}$$

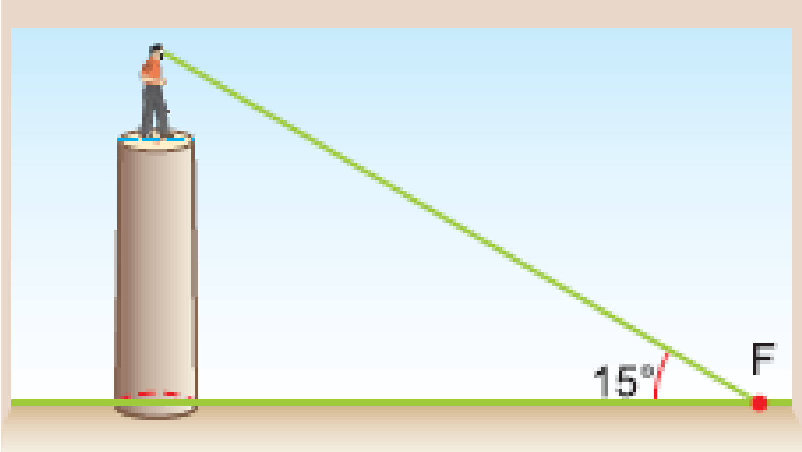
$$b)\ cos(a-b) = cosa.cosb + sena.senb \rightarrow cos(a-b) = \frac{4}{5}.\frac{5}{13} + \frac{3}{5}.\frac{12}{13} \rightarrow cos(a-b) = \frac{20}{65} + \frac{36}{65} = \frac{56}{65}$$

$$c)\ sen(2a) = 2.sena.cosa \rightarrow sen(2a) = 2.\frac{3}{5}.\frac{4}{5} \rightarrow sen(2a) = \frac{24}{25}$$

$$d)\ cos(2b) = cos^2b - sen^2b \rightarrow cos(2b) = \left(\frac{5}{13}\right)^2 - \left(\frac{12}{13}\right)^2 \rightarrow cos(2b) = \frac{25}{169} - \frac{144}{169} \rightarrow cos(2b) = -\frac{119}{169}$$

$$e)\ tg(2a) = \frac{2tga}{1 - tg^2a} \rightarrow tg(2a) = \frac{\frac{2.3}{4}}{1 - \left(\frac{3}{4}\right)^2} \rightarrow tg(2a) = \frac{\frac{6}{4}}{1 - \frac{9}{16}} \rightarrow tg(2a) = \frac{\frac{6}{4}}{\frac{7}{16}} \rightarrow tg(2a) = \frac{6}{4}.\frac{16}{7} = \frac{24}{7}$$

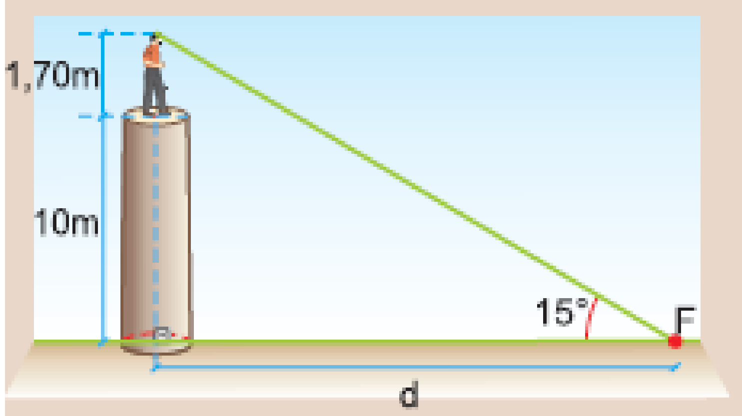
7) **(MODELO ENEM)** – Em uma região plana de um parque estadual, um guarda florestal trabalha no alto de uma torre cilíndrica de madeira de 10 m de altura. Em um dado momento, o guarda, em pé no centro de seu posto de observação, vê um foco de incêndio próximo à torre, no plano do chão, sob um ângulo de 15° em relação à horizontal. Sabe-se que a altura do guarda é 1,70 metros.



Calcular, aproximadamente, a distância do foco ao centro da base da torre, em metros.

Obs. Use $\sqrt{3} = 1{,}7$ antes de racionalizar

a) 31 b) 33 c) 35 d) 41 e) 45



$$tg15° = \frac{10 + 1,70}{d} \rightarrow \text{tg}(60° - 45°) = \frac{11,70}{d} \rightarrow \frac{tg60° - tg45°}{1 + tg60°.tg45°} = \frac{11,70}{d}$$

$$\frac{\sqrt{3} - 1}{1 + \sqrt{3}.1} = \frac{11,70}{d} \rightarrow \frac{1,7 - 1}{1 + 1,7} = \frac{11,70}{d} \rightarrow \frac{0,7}{2,7} = \frac{11,70}{d} \rightarrow 0,7d = 31,59 \rightarrow d = \frac{31,59}{0,7} \rightarrow d \cong 45,12\,m$$

$GABARITO: E$

8) Calcular tg (2x) sabendo que tg x = 3.

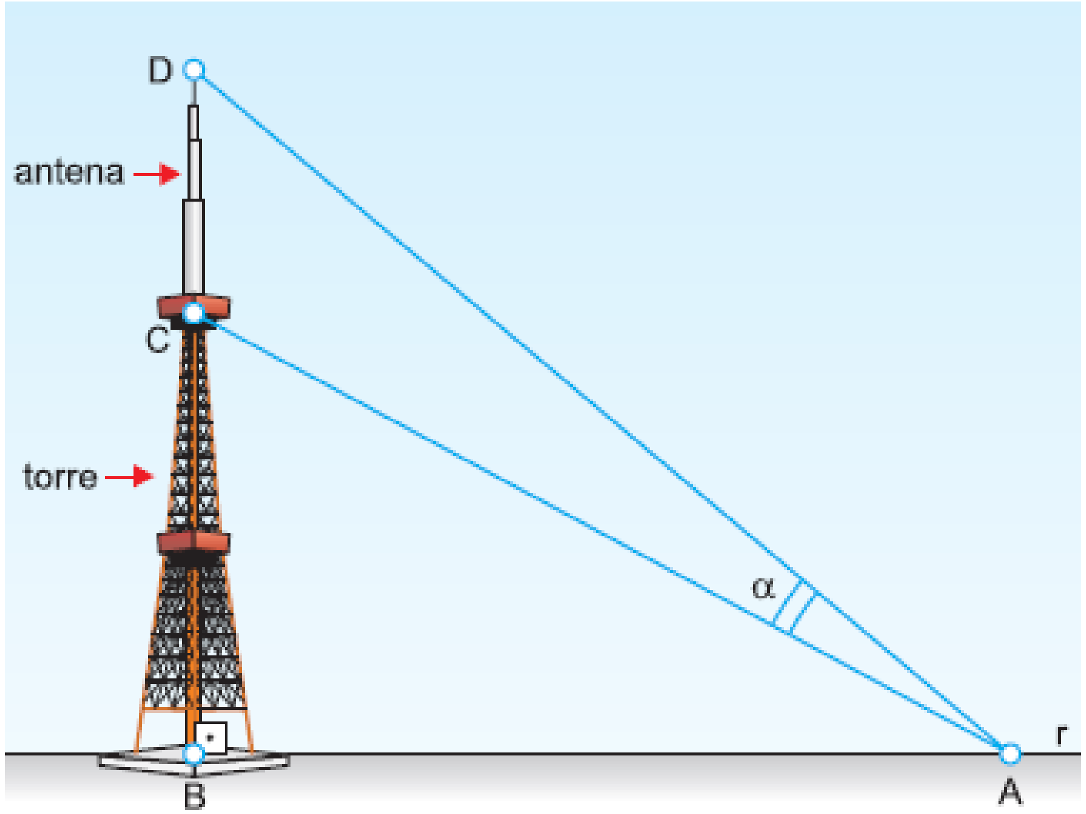
$$tg(2x) = \frac{2.tgx}{1 - tg^2x} \to tg(2x) = \frac{2.3}{1 - 3^2} \to tg(2x) = \frac{6}{-8} \to tg(2x) = -\frac{6}{8} = -\frac{3}{4}$$

9) $Sabendo\ que\ sena + cosa = \frac{1}{2}, encontre\ sen(2a).$

$$(sena + cosa)^2 = \left(\frac{1}{2}\right)^2 \to sen^2a + 2.sena.cosa + cos^2a = \frac{1}{4} \to 1 + sen(2a) = \frac{1}{4}$$

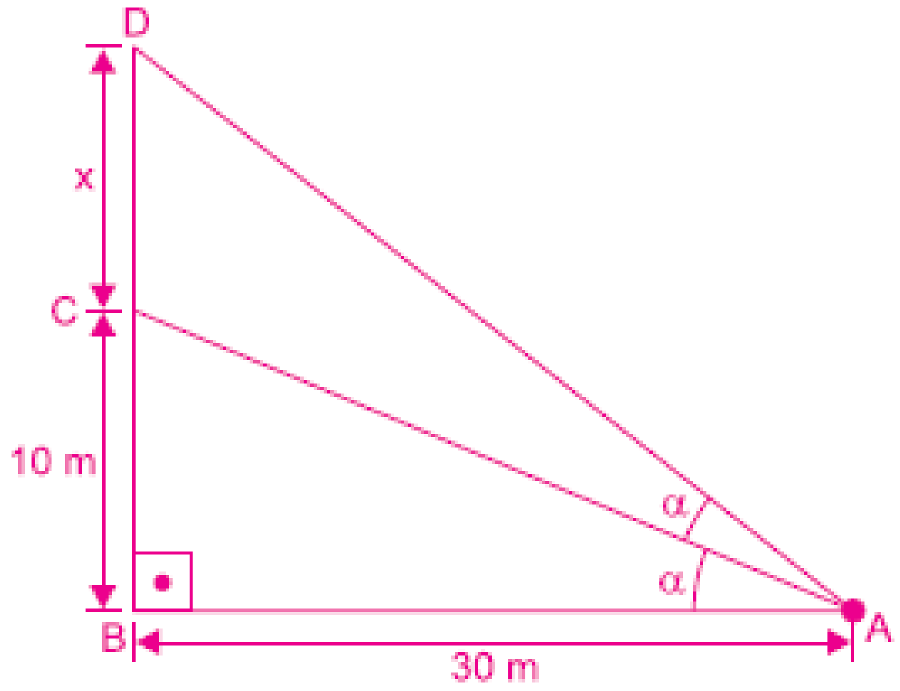
$$sen(2a) = \frac{1}{4} - 1 \to sen(2a) = -\frac{3}{4}$$

10) **(MODELO ENEM)** – Na figura abaixo, o segmento BC representa uma torre metálica vertical com 10 metros de altura, sobre a qual está fixada uma antena transmissora de sinais de uma estação de rádio FM, também vertical, com x metros de comprimento.



A reta **r** é uma das retas do plano do chão, que passa pela base B da torre. Sabe-se que o ângulo CÂD, no qual A é um ponto de **r**, distante 30 m de B, tem medida α. Qual será o tamanho da antena CD, se o ângulo CÂB também tiver a medida α?

a) 20 m.   b) 18 m.   c) 17,5 m.   d) 14 m.   e) 12,5 m.



$$tg\alpha = \frac{10}{30} \rightarrow tg\alpha = \frac{1}{3}$$

$$tg(2\alpha) = \frac{10 + x}{30}$$

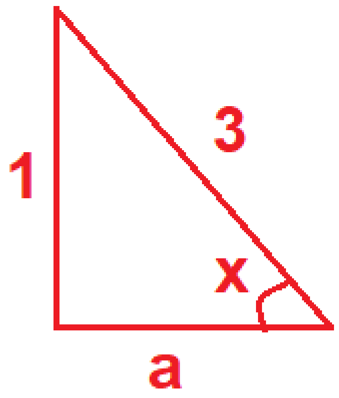
$$tg(2\alpha) = \frac{2tg\alpha}{1 - tg^2\alpha} \rightarrow tg(2\alpha) = \frac{2.\frac{1}{3}}{1 - \left(\frac{1}{3}\right)^2} \rightarrow tg(2\alpha) = \frac{\frac{2}{3}}{1 - \frac{1}{9}} \rightarrow tg(2\alpha) = \frac{\frac{2}{3}}{\frac{8}{9}} \rightarrow tg(2\alpha) = \frac{2}{3}.\frac{9}{8} = \frac{3}{4}$$

$$\frac{3}{4} = \frac{10 + x}{30} \rightarrow 40 + 4x = 90 \rightarrow 4x = 50 \rightarrow x = \frac{50}{4} \rightarrow x = 12,5\ m$$

$GABARITO: E$

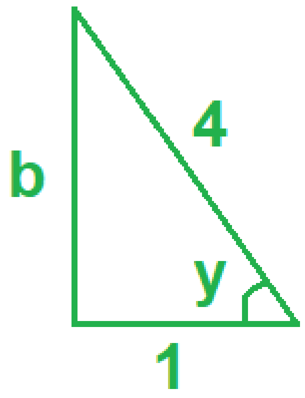
$11)\ Sabendo\ que\ senx = \frac{1}{3}, cosy = \frac{1}{4}, x \in 2°\ quadrante\ e\ y \in 4°\ quadrante, determine:$

$a)\ sen(x+y); b)\ \cos(x-y)\ ; c)\ sen(2x); d)\ \text{tg}(2x)$



$$3^2 = 1^2 + a^2 \rightarrow 9 = 1 + a^2 \rightarrow a^2 = 8 \rightarrow a = \sqrt{8} \rightarrow a = 2\sqrt{2}$$

$$\begin{cases} senx = \frac{1}{3} \\ cosx = -\frac{2\sqrt{2}}{3} \\ tgx = -\frac{1}{2\sqrt{2}} x \frac{\sqrt{2}}{\sqrt{2}} = -\frac{\sqrt{2}}{4} \end{cases}$$



$$4^2 = 1^2 + b^2 \rightarrow 16 = 1 + b^2 \rightarrow b^2 = 15 \rightarrow b = \sqrt{15}$$

$$\begin{cases} cosy = \frac{1}{4} \\ seny = -\frac{\sqrt{15}}{4} \end{cases}$$

$$\begin{cases} senx = \dfrac{1}{3} \\ cosx = -\dfrac{2\sqrt{2}}{3} \\ tgx = -\dfrac{1}{2\sqrt{2}} x \dfrac{\sqrt{2}}{\sqrt{2}} = -\dfrac{\sqrt{2}}{4} \end{cases}$$

$$\begin{cases} cosy = \dfrac{1}{4} \\ seny = -\dfrac{\sqrt{15}}{4} \end{cases}$$

$$a)\ sen(x+y) = senx.cosy + seny.cosx \rightarrow sen(x+y) = \frac{1}{3}.\frac{1}{4} + \left(-\frac{\sqrt{15}}{4}\right).\left(-\frac{2\sqrt{2}}{3}\right)$$

$$sen(x+y) = \frac{1}{12} + \frac{2\sqrt{30}}{12} = \frac{1+2\sqrt{30}}{12}$$

$$b)\ cos(x-y) = cosx.cosy + senx.seny \rightarrow cos(x-y) = -\frac{2\sqrt{2}}{3}.\frac{1}{4} + \frac{1}{3}.\left(-\frac{\sqrt{15}}{4}\right)$$

$$cos(x-y) = -\frac{2\sqrt{2}}{12} - \frac{\sqrt{15}}{12} \rightarrow cos(x-y) = \frac{-2\sqrt{2}-\sqrt{15}}{12}$$

$$\begin{cases} senx = \dfrac{1}{3} \\ cosx = -\dfrac{2\sqrt{2}}{3} \\ tgx = -\dfrac{1}{2\sqrt{2}} x \dfrac{\sqrt{2}}{\sqrt{2}} = -\dfrac{\sqrt{2}}{4} \end{cases}$$

$$\begin{cases} cosy = \dfrac{1}{4} \\ seny = -\dfrac{\sqrt{15}}{4} \end{cases}$$

c) $sen(2x) = 2.senx.cosx \rightarrow sen(2x) = 2.\dfrac{1}{3}.\left(-\dfrac{2\sqrt{2}}{3}\right) \rightarrow sen(2x) = -\dfrac{4\sqrt{2}}{9}$

d) $tg(2x) = \dfrac{2tgx}{1 - tg^2x} \rightarrow tg(2x) = \dfrac{2.\left(-\dfrac{\sqrt{2}}{4}\right)}{1 - \left(-\dfrac{\sqrt{2}}{4}\right)^2} \rightarrow tg(2x) = \dfrac{-\dfrac{\sqrt{2}}{2}}{1 - \left(\dfrac{2}{16}\right)} \rightarrow tg(2x) = \dfrac{-\dfrac{\sqrt{2}}{2}}{1 - \dfrac{1}{8}}$

$tg(2x) = \dfrac{-\dfrac{\sqrt{2}}{2}}{\dfrac{7}{8}} \rightarrow tg(2x) = -\dfrac{\sqrt{2}}{2}.\dfrac{8}{7} \rightarrow tg(2x) = -\dfrac{4\sqrt{2}}{7}$

12) *Sabendo que* $sen53° \cong 0{,}8$, *calcule o valor aproximado de* $sen23°$.

$a)\ 0{,}2.\sqrt{2} - 1$ $b)\ 0{,}4.\sqrt{3} - 0{,}3$ $c)\ 0{,}5.\sqrt{2} - 0{,}2$ $d)\ 0{,}6.\sqrt{3} - 0{,}3$

$$\boldsymbol{sen23° = sen(53° - 30°) \rightarrow sen23° = sen53°.cos30° - sen30°.cos53°}$$

$$\boldsymbol{sen^2 53° + cos^2 53° = 1 \rightarrow (0,8)^2 + cos^2 53° = 1 \rightarrow 0,64 + cos^2 53° = 1 \rightarrow cos^2 53° = 0,36}$$

$$\boldsymbol{cos53° = \sqrt{0,36} \rightarrow cos53° = 0,6}$$

$$\boldsymbol{sen23° = 0,8.\frac{\sqrt{3}}{2} - \frac{1}{2}.0,6 \rightarrow sen23° = 0,4.\sqrt{3} - 0,3}$$

GABARITO: B